Reflexions on True North, Large Dials and the Apparent Sun

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It is always very satisfying if one can establish true North by the traditional equal-angle method but sadly in Britain the opportunities for this can be rare indeed. The more so if a large dial has to be set up since the inevitable constraints of builders, architects and long distances all contrive to require the process to be completed on or around a particular date and, for some reason on a date that is usually nearer to midwinter than midsummer with all of the bad weather implications that that brings! It is therefore usually necessary to fall back upon higher technology and there seem to be as many different approaches to this as there are dial designers.

I take no credit for the novelty of this approach to the problem of establishing true North to a precision and with an accuracy necessary for the setting up of a large dial but I have not seen it set down in this particular way and thought it might prove to be of interest to others who might be faced with the problem of delineation and construction of a large dial. Of course the details have to be tailored to the dial type but here I refer to the problem as it relates to a 12metre diameter horizontal dial erected as part of the Millennium redevelopment of a Northumbrian public square[1].

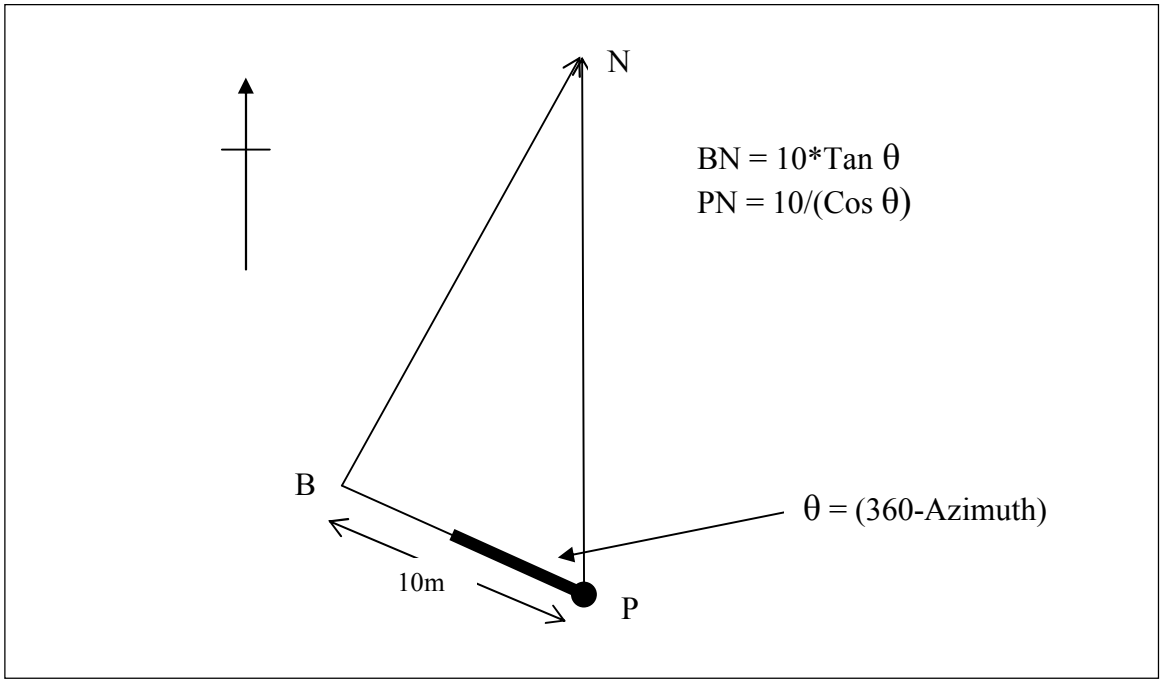

Figure 1

After the land has been cleared and is horizontal and firm, a vertical cylindrical pole some two or more metres in height should be firmly erected and set in concrete just South of the dial centre. A scaffolding pole is usually sufficient although the diameter should be chosen so that at the time of the measurements there will be a sufficient umbra within which to mark the centre of the shadow on a few convenient sunny moments during the day and at a moment noted each time to the exact minute for example by use of a mobile phone and the 'speaking clock'. Referring to the plan view in Figure 1, the pole is represented at P. A particular distance (say ten metres) is then measured out from the pole's centre along the line of the shadow to establish point B. The distances BN and PN of the right angled triangle PBN are calculated from a knowledge of the sun's azimuth at the time of the reading (here azimuth from the North has been assumed). Thus two arcs can be drawn and a marker inserted into the ground at their intersection N. The line PN then represents one measure of the line of true North. A mean of several

such lines then gives the best line on which to orient the dial. The system is simple, requires no angles to be measured or constructed and merely requires linear measurements to be made.

Considerable precision may be obtained by this method. The main difficulties stem from the problems of

- locating the centre of the shadow's umbra with sufficient precision;
- properly measuring distances from the centre of the pole, given that the shadow length may not be very long,
- possible errors on uneven ground from measuring out of the horizontal plane; and
- an inaccurate estimate of the *apparent* azimuth of the sun.

The first three of these are problems of precision well known to most diallists and each will have their own approaches to them. It is this last difficulty that will now be examined in more detail since it, of all the sources of error, can less easily be reduced by operational practice. It is an issue of accuracy of calculation rather than of precision of measurement.

It might be thought that it is only of importance to reduce each error to a figure comparable with the others. However errors in dialling have an unreasonable habit of becoming cumulative. As a consequence it is always good practice to pay attention to all opportunities for error and aim to keep every one to as low a level as may be sensibly possible.

The estimation of the sun's position is a case in point. In determining North by the above method and, when using a ten metre base, the length of the arc from P to N can frequently be of the order of 15 metres and a marker can easily be placed to within 1 cm at such a distance. This corresponds to an angle of precision of just over 2 minutes of arc or 0.04 degree. Thus we may say that if possible we would like to use values of the sun's azimuth accurate to this value or better. It is easy to show that a similar accuracy might be required of the sun's altitude if the proposed dial is to use a nodus to show declination.

Astronomers calculate the sun's position to the greatest accuracy by using the VSOP 87[2] theory. However this involves over 2400 terms and is wholly impracticable for normal use by diallists. What is more to the point though is that it delivers the true astronomical position of the sun but what is needed for North determination is the *apparent* position of the sun at the dial's location. Jean Meeus[3] has derived an abbreviated set of terms from the VSOP87 theory and developed corrected conversions to local azimuth and altitude. Despite their being abbreviated, these terms nevertheless only involve a discrepancy of not more than 1 arc-sec between the years -2000 to +6000. Even after making the necessary corrections for refraction and parallax the results are still highly accurate for times not close to sunrise and sunset. If the VSOP87 theory can be said to be the Platinum standard of reference then this abbreviated set of terms can be regarded as the Gold Standard for diallists.

Fortunately for most of us, this 'Gold Standard' theory is available to all via a web site[4] which was set up specifically as an aid for sundial construction. The *Solar Calculator* makes use of Jean Meeus' formulae for nearly all its calculations including date conversion, equation of time, solar coordinates, horizontal coordinates, correction for atmospheric refraction etc. It is thus among the best available at this time for dial calculations.

But what really is the benefit of going to such lengths when other approaches might be easier to use? After all, the *Solar Calculator* does require that connexion be made to the Internet for it to be used and this is not always convenient. Some diallists might instead prefer to use software that already runs offline on their PC or laptop without the need for an Internet connexion.

Of the many such that are available, the NASS *Dialist's Companion*[5] (v1.1b, a Shareware product) and *WinEphem*[6] (v1.9, Freeware) are probably two of the best known. Of course one might also consider using the equations and tables given in Waugh's book[7] within a spreadsheet of one's own design..

It is interesting to make a simple comparison of the performances of these systems against those of the *Solar Calculator* as the reference. By way of illustration rather than as an exhaustive comparison, this has been achieved here by plotting the sun's position for the whole of 2003 for Lat 52º:30'N, Long 00º:00'W at 30 day intervals from the 15th January and using the same time of 10:00am GMT for each point. A time of 10:00am was chosen so as to

generally avoid much need for reliance on corrections to altitude for refraction.

The *Dialist's Companion*, which also uses some of Meeus' equations and *WinEphem*, based on P Duffett-Smith's earlier work[8], are both standalone products that readily give solar azimuth and altitude data. The *Dialist's Companion* of course does very, very much more for the diallist than *WinEphem* but the latter is additionally useful as an ephemeris for the planets and some stars.

Figures 2 and 3 below show the respective differences of these two products from the *Solar Calculator* over the year 2003.

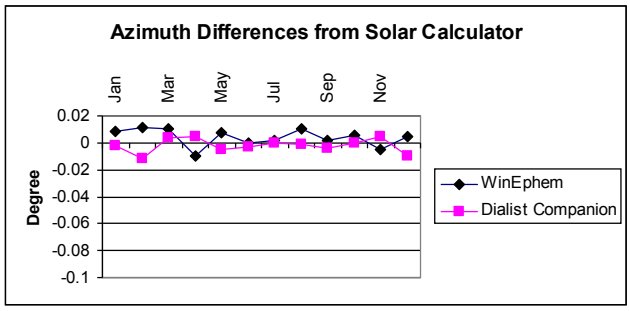

Figure 2

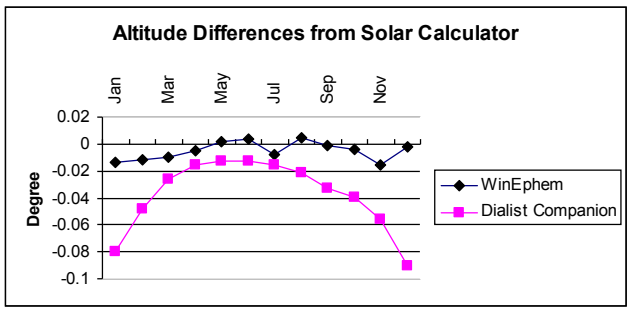

Figure 3

Both *WinEphem* and the *Dialist's Companion* have excellent correlation in azimuth and hence can most certainly be used for the establishment of true North to the accuracy mentioned above. Where altitude is concerned however, there appears to be a greater variation from the *Dialist's Companion* in the winter months which may be a consequence of the fact that this product only makes its (optional) refraction adjustments to the total time correction and not to its calculations of solar altitude.

Perhaps one of the main disadvantages of both *WinEphem* and the *Dialist's Companion* for the establishment of true North is that they are only easily useful for noting a small number of individual measurements, one at a time. This is especially the case with *WinEphem* since it yields its data only in degrees and minutes rather than decimals of degrees.

In attempting to lay out true North for a large dial in difficult conditions it can often be useful to be able to prepare beforehand a listing of the positions of the sun at many times of the day. Indeed in situations where adverse weather can be foreseen, I have sometimes found it necessary to prepare tables of the sun's position – together with the associated arc lengths for determination of true north - for every minute of one, or even more, working days ahead.

The *Solar Calculator* in its Text (rather than its HTML) output mode, is particularly suited to this approach as, of course, would be the development of one's own spreadsheet using the equations given in Waugh. However as may be seen from the plots below, Waugh's simplistic formulae yield considerable differences from those of the *Solar Calculator*. It is sometimes asserted that better accuracy can be obtained from Waugh's equations if the instantaneous values for the Equation of Time and/or the sun's declination at the time of the observation are introduced rather than the single 'daily' values given in Waugh's tables. The graphs in Figures 4 and 5 show that this is only marginally the case and that no matter what approach might be used, Waugh's formulae can yield an error of up to a degree in both azimuth and altitude at certain times of the year.

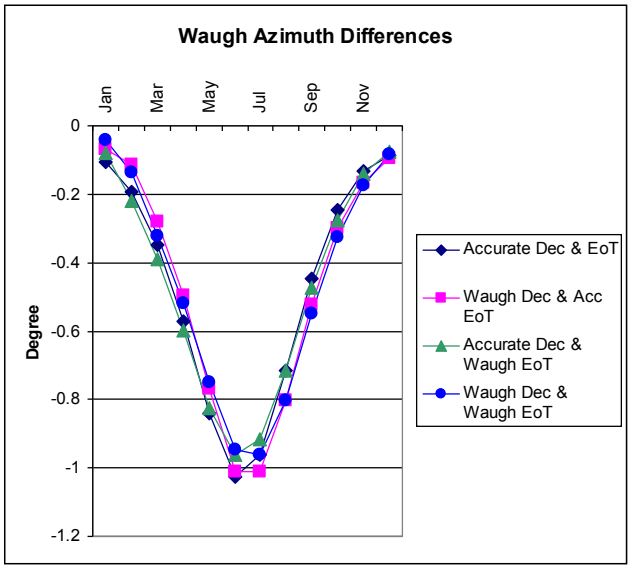

Figure 4

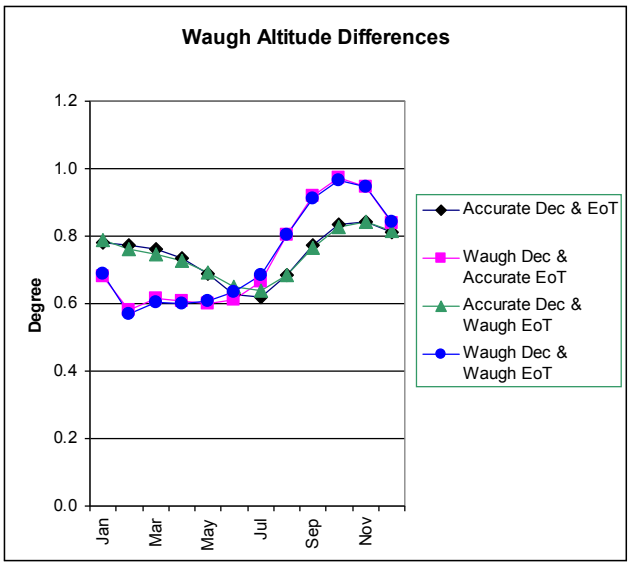

Figure 5

All in all the use of the *Solar Calculator* is to be preferred for the development of pre-prepared tables of the sun's position and of the necessary meridian construction arcs if one is trying to set up a large dial.

We have discussed the precision of measurement but we have not been able to do other than take on trust the accuracy of the calculation of the sun's coordinates using the *Solar Calculator*. The real success of this approach can only be judged once the dial has been installed and all of the errors, not only including those of delineation but also those even less controllable ones of construction, have been combined.

So how did it all work out for the Northumberland Dial? Readers may judge for themselves from the fact that when using the approach described here and with data from *Solar Calculator,* true North was established successfully one wet November day on the basis of only two shadow measurements. Indeed, both of those had to be taken within 20 minutes of each other because the sun was only to be seen for a few minutes in a three day period. In the event the dial has proved to be accurate to 15 seconds. A tribute not only to the accuracy of *Solar Calculator* but perhaps even more so to the builders who were responsible for the site construction and for the installation of the gnomon.

References:

[1] P Powers. 'The Amble Dial', at http://www.dunelm.org.uk/homepages?patrick_powers. Then link to 'Sundials' and 'Amble'
[2] P Bretagnon, *Variations Séculaires des Orbites Planétaires, Bureau des Longitudes de Paris, 1987*
[3] J Meeus, *Astronomical Algorithms,* Willmann-Bell, Inc, Richmond, VA, Second Edition, March 2000
[4] LJ Coletti, 'Great Circle Studio's Solar Calculator', at http://www.gcstudio.com/cgi-bin/sunpage. Text Mode. 1998
[5] R Terwilliger and F Sawyer, 'The Dialist's Companion', (v1.1b), 1996, at http://sundials.org/.
[6] TM James, 'WinEphem', *(v1.09)*, at http://www.geocities.com/tmarkjames/WinEphem.html
[7] AE Waugh, *Sundials their Theory and Construction*, Dover Publications, Inc New York, 1973. p139 & Tables A1 & A2
[8] P Duffett-Smith, *Astronomy with your Personal Computer,* Cambridge University Press, 1985