Finding the First Point of Aries from first principles…

Question

Pretend it is 1580. How would Tycho Brahe have located the First Point of Aries? Or for that matter Claudius Ptolemy back in 120 AD?

Method 1

Here's how you would find the approximate location of the First Point of Aries relative to the stars. (If you want greater accuracy, you have to apply increasingly sophisticated methods.)

Given the fact that you have a telescope or observing tube with accurate angle markings (such as Tycho Brahe's measuring instruments), you would do the following:

1) Prior to building the measuring instruments, you must accurately establish the cardinal directions. That can be done by observing the north star over the course of a night (assuming you live in the northern hemisphere).

2) You also need to establish your latitude, which is also done by observing the north star.

3) The Julian calendar was long in use at the time of Tycho. You need to establish the day of the autumnal equinox. You can do this by observing the sun every day at sunset, and determining on what day of the year (after summer) it is closest to due west. (Of course, there are minor inaccuracies depending on refraction and the definition of sunset.)

4) On the night of the autumnal equinox, you use your clock to observe the sky six hours after sunset. You aim your observing instrument due south, at an altitude of 90 degrees minus your latitude. You are now looking at the First Point of Aries, and can measure its position with respect to nearby stars.

To improve accuracy a bit, you could do the following: Rather than observing the sky six hours after sunset (which is affected by refraction and the definition of sunset), observe it midway between sunset and sunrise. This requires you to do the following:
1) Record the time of sunset, T1.
2) Start observing the sky (due south, at altitude 90-latitude) several minutes before T1+6 hours until several minutes after T1+6 hours.
3) Record the time of sunrise T2.
4) Determine which of your observations was made most closely midway between sunset and sunrise. That eliminates many (but not all) of the systematic errors associated with the definition of sunset.

Method 2.

Here's another approximate method: Suppose you have determined the equinox dates. If you wait enough years, there will be an eclipse of the moon on either of these dates. During a lunar eclipse, the moon is always about 180 degrees from the sun. If the eclipse happens on the night of the autumnal equinox, the moon is approximately at the First Point of Aries, and you can observe its position relative to nearby stars. If the eclipse happens on the night of the vernal equinox, the moon is approximately 180 degrees from the First Point of Aries. Similarly, if the eclipse occurs on one of the solstice dates, the moon is about 90 degrees from the First Point of Aries.

This method can be generalized as follows to avoid the problem of having to wait many years for an equinox eclipse:

1) First, you determine the location of the celestial equator in the sky. Throughout the night (and at different times in the year), you observe due south at an altitude of 90-latitude. This lets you determine the path of the celestial equator relative to the stars.

2) Every time there's an eclipse of the moon (particularly a total eclipse), you observe the location of the eclipsed moon relative to nearby stars. By observing many eclipses and drawing a "best fit" through the lunar locations, you get the approximate path of the ecliptic relative to the stars. (The ecliptic is the path of the sun, and the eclipsed moon is always very close to the ecliptic.)

3) You create a star map and draw both the ecliptic and equator. The intersections are the First Point of Aries and its diametrically opposite point.

Method 3

Alternatively, instead of waiting for eclipses, you could plot the location of the moon every night as a "scatter plot" on a star map. After about 18.6 years, you'd find that the locations form a band about 10 degrees wide, and the center of this band is the ecliptic. Once again, the First Point of Aries is the intersection of the ecliptic and the celestial equator.

(18.6 years is the period of the regression of the lunar nodes. You could obtain a result in less time, but the 10-degree-wide band would become fully populated after observations spanning 18.6 years or more.)